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262. Proposed by NELSON L. RORAY, Utica, New York.

In a regular pentagon, show that  $\frac{\text{diagonal}}{\text{side}} = \frac{2 \text{ apothem}}{\text{radius}} = \frac{5 + \sqrt{5}}{2\sqrt{5}}$ .

Solution by P. S. BERG, Larimore, N. Dak.

In a regular pentagon the diagonal is  $\frac{R}{2}\sqrt{10+2\sqrt{5}}$ , and the side is  $\frac{R}{2}\sqrt{10-2\sqrt{5}}$ .

$$\frac{\frac{R}{2}\sqrt{10+2\sqrt{5}}}{\frac{R}{2}\sqrt{10-2\sqrt{5}}} = \frac{\frac{R}{2}\sqrt{6+2\sqrt{5}}}{R}.$$

But  $\frac{R}{2}\sqrt{6+2\sqrt{5}} = 2 \text{ apothem}$ , and  $R = \text{radius}$ .

$$\frac{\frac{R}{2}\sqrt{6+2\sqrt{5}}}{R} = \frac{\sqrt{6+2\sqrt{5}}}{2} = \frac{1}{2} \cdot \frac{\sqrt{(30+10\sqrt{5})}}{\sqrt{5}} = \frac{1}{2} \cdot \frac{5+\sqrt{5}}{\sqrt{5}}.$$

Also solved by G. B. M. Zerr, R. D. Carmichael, A. H. Holmes, and L. E. Newcomb.

## MECHANICS.

180. Proposed by EDWIN L. RICH, Lehigh University, South Bethlehem, Pa.

If a body is projected into the air, and the resistance of the air varies as the square of the velocity; required the equation of the curve. [From De Volson Wood's *Analytical Mechanics*, problem 10, p. 179.]

Solution by G. B. M. ZERR.

Let the direction of projection be in the  $xy$  plane. Let  $v = \text{velocity of projection}$ ,  $g = \text{acceleration of gravity}$ ,  $\alpha = \text{angle of projection}$ ,  $k(ds/dt)^2 = \text{resistance at any time } t$ ,  $\phi = \text{inclination of direction of motion to the horizon at any time } t$ . The  $x$ - and  $y$ - components of resistance are, respectively,  $k \frac{ds}{dt} \frac{dx}{dt}$ , and  $k \frac{ds}{dt} \frac{dy}{dt}$ .

Resolving horizontally and vertically, the equations of motion are

$$\begin{aligned} d^2x/dt^2 &= -k(ds/dt)(dx/dt) \dots (1), \\ dy^2/dt^2 &= -g - k(ds/dt)(dy/dt) \dots (2). \end{aligned}$$

From (1),  $d(dx/dt)/(dx/dt) = -kds$ .

$\therefore \log[(dx/dt)/v \cos \alpha] = -ks$ . When  $t=0$ ,  $dx/dt = v \cos \alpha$ .

$\therefore dx/dt = v \cos \alpha e^{-ks} = u \dots (3)$ .

Resolving in the direction of the tangent and normal,

$$d^2s/dt^2 = -g \sin \phi - k(ds/dt)^2 \dots (4).$$

Let  $v_1$  = velocity of particle at any point; then (4) becomes

$$d^2s/dt^2 = -g \sin \phi - k v_1^2 \dots (5),$$

$$v_1^2/\rho = g \cos \phi \dots (6).$$

But  $u = v_1 \cos \phi$  and  $\rho = -ds/d\phi$ .  $\therefore$  (5) and (6) become

$$du/dt = -k v_1^2 \cos \phi \dots (7),$$

$$v_1 (d\phi/dt) = -g \cos \phi \dots (8).$$

$$\therefore \frac{du}{d\phi} = \frac{k v_1^3}{g} = \frac{k}{g} u^3 \sec^3 \phi \dots (9).$$

At the origin,  $u = v \cos \alpha$ . Integrating (9) we get

$$\frac{1}{v^2 \cos^2 \alpha} - \frac{1}{u^2} = \frac{k}{g} (A_\phi - A_\alpha) \dots (10).$$

$$\text{Where } A_\phi = \int_0^\phi 2 \sec^3 \phi d\phi, \quad A_\alpha - A_\phi = 2 \int_\phi^\alpha \sec^3 \phi d\phi$$

$$= \tan \alpha \sec \alpha - \tan \phi \sec \phi + \log \left( \frac{\tan \alpha + \sec \alpha}{\tan \phi + \sec \phi} \right).$$

Substituting in (10) we get

$$\frac{k}{g} v^2 \cos^2 \alpha \left[ \tan \alpha \sec \alpha - \tan \phi \sec \phi + \log \left( \frac{\tan \alpha + \sec \alpha}{\tan \phi + \sec \phi} \right) \right] = e^{2ks} - 1,$$

for the intrinsic equation to the curve.

#### MISCELLANEOUS.

145. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

Given  $\sin 3\phi + \cos 3\phi = m$ .....(1), and  $\cos \phi - \sin \phi = x$ .....(2), to find  $x$  in terms of  $m$ .

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Since  $3\sin \phi - 4\sin^3 \phi = \sin 3\phi$  and  $-3\cos \phi + 4\cos^3 \phi = \cos 3\phi$ , we have  $4(\cos^3 \phi - \sin^3 \phi) - 3(\cos \phi - \sin \phi) = m$ , or  $(\cos \phi - \sin \phi)[4(\cos^2 \phi + \cos \phi \sin \phi + \sin^2 \phi) - 3] = m$ .

Since  $\cos \phi - \sin \phi = x$ ,  $2\sin \phi \cos \phi = 1 - x^2$  and therefore  $2x^3 - 3x + m = 0$ .

Also solved by A. H. Holmes, and the Proposer.